Online Appendix for: Listen to the market, hear the best policy decision, but don't always choose it.

David Reinstein* and Joon Song†

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1 Online Appendix

1.1 Derivation for case of fixed A_0, no short-selling, condition (20)

The first set of incentive compatibility constraints (23) is equivalent to the following under condition (20):

$$q(\gamma) \geq \frac{-\mathbb{E}\, V(ne|\gamma)}{\mathbb{E}\, V(e|\gamma) - \mathbb{E}\, V(ne|\gamma)} \quad \text{and} \quad q(\gamma) \leq \frac{-\mathbb{E}\, V(ne|\beta)}{\mathbb{E}\, V(e|\beta) - \mathbb{E}\, V(ne|\beta)}.$$

Moreover we get the following with condition (20):

$$\frac{-\mathbb{E}\, V(ne|\gamma)}{\mathbb{E}\, V(e|\gamma) - \mathbb{E}\, V(ne|\gamma)} < \frac{-\mathbb{E}\, V(ne|\beta)}{\mathbb{E}\, V(e|\beta) - \mathbb{E}\, V(ne|\beta)}.$$

Thus, the two constraints become

$$\frac{-\mathbb{E}\, V(ne|\gamma)}{\mathbb{E}\, V(e|\gamma) - \mathbb{E}\, V(ne|\gamma)} \leq q(\gamma) \leq \frac{-\mathbb{E}\, V(ne|\beta)}{\mathbb{E}\, V(e|\beta) - \mathbb{E}\, V(ne|\beta)}.$$

The policymaker wants to increase $q(\gamma)$ as long as the two incentive compatibility constraints are satisfied. Thus, we conclude:

$$q(\gamma) = \frac{-\mathbb{E}\, V(ne|\beta)}{\mathbb{E}\, V(e|\beta) - \mathbb{E}\, V(ne|\beta)} = \frac{1}{\frac{\mathbb{E}\, V(e|\beta)}{-\mathbb{E}\, V(ne|\beta)} + 1}.$$

Note that this value is between 0 and 1 under condition 20.

*Department of Economics, University of Essex, drein@essex.ac.uk

†corresponding author, Department of Economics, Sungkyunkwan University, joonsong.econ@gmail.com

1.2 [Online] Model with short selling

With short selling, there are potentially six distinct ways of implementing truth telling behavior: this can occur through $\mathcal{II}$ choosing any of "buy, do nothing, or short sell" after a good signal, and choosing any *other* action after a bad signal. Each of these implementations requires that four IC constraints be satisfied, as after each signal, one action must be preferred above the other two actions. For tractability we focus on two potential implementations: "buy if γ and short sell if β", and vice-versa".

Set 1 (Buy if and only if the signal is γ):

Here, the PM interprets $i = b$ as $\mathcal{II}$ having received γ. The IC constraints when short selling is allowed are:

$$q(\gamma)\Delta A_1(\gamma) + A_1(ne) - A_0(q(\gamma), q(\beta)) \geq -q(\beta)\Delta A_1(\gamma) - A_1(ne) + A_0(q(\gamma), q(\beta)), \qquad (IC_{11})$$

$$-q(\beta)\Delta A_1(\beta) - A_1(ne) + A_0(q(\gamma), q(\beta)) \geq q(\gamma)\Delta A_1(\beta) + A_1(ne) - A_0(q(\gamma), q(\beta)). \qquad (IC_{12})$$

The first constraint ensures the $\mathcal{II}$ prefers to buy when he gets signal γ; the second constraint ensures he prefers to short sell when he gets signal β.

We show in section B.4 that choosing *doing nothing* is never superior to choosing *buy* in the current context.

We rearrange IC_{11} for intuition (the intuition for IC_{12} is similar):

$$\begin{aligned} &\underbrace{q(\gamma)\Delta A_1(\gamma)}_{(i)} - \underbrace{(q(\gamma)\Delta A_1(\gamma)P(\gamma) + q(\beta)\Delta A_1(\beta)P(\beta))}_{(ii)} \\ &\geq \underbrace{-q(\beta)\Delta A_1(\gamma)}_{(i)} + \underbrace{(q(\gamma)\Delta A_1(\gamma)P(\gamma) + q(\beta)\Delta A_1(\beta)P(\beta))}_{(ii)} \end{aligned}$$

On each side of the above inequality, (i) represents the $\mathcal{II}$'s expectation of the gain (or loss) in the asset's value given the signal he is sending, and (ii) is the ex-ante expected gain (or loss) in the asset's value.

Note that if short-selling is not allowed, the right-hand side will be zero.

Inequalities IC_{11} and IC_{12} can also be written as:

$$\Delta A_1(\gamma)\Big[q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big)+q(\gamma)(1-2P(\gamma)))\Big]\geq 0, \qquad (IC'_{11})$$

$$\Delta A_1(\beta)\Big[q(\gamma)\Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big)+q(\beta)(1-2P(\beta)))\Big]\leq 0. \qquad (IC'_{12})$$

Set 2 (Buy if and only if the signal is β):

Here, the PM interprets $i = sh$ as $\mathcal{II}$ having received γ. The incentive compatibility constraints when short sale is allowed are:

$$-q(\gamma)\Delta A_1(\gamma)-A_1(ne)+A_0(q(\gamma),q(\beta))\geq q(\beta)\Delta A_1(\gamma)+A_1(ne)-A_0(q(\gamma),q(\beta)), \qquad (IC_{21})$$

$$q(\beta)\Delta A_1(\beta)+A_1(ne)-A_0(q(\gamma),q(\beta))\geq -q(\gamma)\Delta A_1(\beta)-A_1(ne)+A_0(q(\gamma),q(\beta)), \qquad (IC_{22})$$

The first constraint ensures that $\mathcal{II}(\gamma)$ prefers buying to short selling; the second constraint ensures that $\mathcal{II}(\beta)$ prefers short selling to buying. Without short-selling, the left (right) side of the first (second) inequality is zero.

Again, we show in section B.4 that choosing *do nothing* is never superior to choosing *buy* in the current context.

Sets 1 and 2 are precisely the converse of each other.

$$\underbrace{[q(\gamma)+q(\beta)]\Delta A_1(\gamma)}_{(a)}\underbrace{-2\mathbb{E}(\Delta A_1)}_{(b)}\geq 0\geq \underbrace{[q(\gamma)+q(\beta)]\Delta A_1(\beta)}_{(a)}\underbrace{-2\mathbb{E}(\Delta A_1)}_{(b)}, \qquad (1)$$

where $\mathbb{E}(\Delta A_1) := A_0 - A_1(ne)$.

1.2.1 Further simplification of the two sets of incentive compatibility constraints

With $q(\gamma) = 1$, we can rewrite the second constraint of Set 1 as:

$$\begin{aligned}
& P(\gamma)\Delta A_1(\gamma)\geq \Delta A_1(\beta)-q(\beta)P(\beta)\Delta A_1(\gamma)\\
\Leftrightarrow\quad & q(\beta)P(\beta)\Delta A_1(\beta)\geq(\Delta A_1(\beta)-P(\gamma)\Delta A_1(\gamma))=(\Delta A_1(\beta)-(1-P(\beta))\Delta A_1(\gamma))\\
& q(\beta)\Delta A_1(\beta)\geq\Big(\frac{\Delta A_1(\beta)-\Delta A_1(\gamma)}{P(\beta)}+\Delta A_1(\gamma)\Big)
\end{aligned}$$

Thus Set 1 is combined into:

$$\Delta A_1(\gamma) \geq \Delta A_1(\beta)q(\beta) \geq \Big[\frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{P(\gamma)} + \Delta A_1(\gamma)\Big], \tag{2}$$

and Set 2 is similarly combined into

$$\Delta A_1(\beta)q(\beta) \geq \Delta A_1(\gamma) \geq \Big[\frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{P(\beta)} + \Delta A_1(\beta)\Big]q(\beta). \tag{3}$$

1.3 [Online] Derivations, allowing short-selling, cases 1-6

Case (i), "A Treat": $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$, *i.e., the asset's value increases when the policy is executed, more so with the good signal.*

Proposition 8 implies that Set 1 is relevant, and Proposition 6 implies $q(\gamma) = 1$. Since $\Delta A_1(\gamma) > 0$ and $\Delta A_1(\beta) > 0$, we can rewrite IC'_{11} and IC'_{12} substituting these out:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \geq 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \leq 0.$$

[Condition 1: $P(\gamma) = \frac{1}{2} = P(\beta)$] Then the two constraints become:

$$q(\beta)\Big(1 - \frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) \geq 0, \quad \Big(1 - \frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) \leq 0.$$

Thus we can reduce $q(\beta)$ to zero, and the incentive-constrained optimal policy achieves the first best, $\langle q(\gamma) = 1, q(\beta) = 0\rangle$.

[Condition 2: $P(\gamma) > \frac{1}{2} > P(\beta)$] Under this condition the above is simplified into:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \geq q(\beta)$$

Both (a) and (b) are larger than 1. Since the PM wants to decrease $q(\beta)$ as much as she can, the only relevant (i.e., binding) constraint is the first one. Thus we conclude:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

[Condition 3: $P(\gamma) < \frac{1}{2} < P(\beta)$] Then the ICs are simplified into:

$$q(\beta)\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)} \leq 1, \quad \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} \leq q(\beta)$$

The first constraint is irrelevant irrespective of the sign of $\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}$ as the PM wants to minimize $q(\beta)$ (note that $q(\beta) = 0$ trivially satisfies the first constraint). If $\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}$ is negative, then the second constraint is also irrelevant; the optimal solution is $\langle q(\gamma) = 1, q(\beta) = 0\rangle$. If $\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}$ is positive, it is smaller than 1. Then the second constraint is relevant, i.e., the second constraint binds, and we derive the optimal solution $\left\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right\rangle$.

To summarize, the optimal policy interprets the $\mathcal{II}$'s buying as signal γ and his not buying as signal β. The optimal incentive-constrained optimal policy is:

$$\text{For } P(\gamma) < 1/2 < P(\beta), \quad \left\langle q(\gamma) = 1, q(\beta) = \max\left(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right)\right\rangle,$$
$$\text{for } P(\gamma) > 1/2 > P(\beta), \quad \left\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle,$$
$$\text{and for } P(\gamma) = 1/2 = P(\beta), \quad \langle q(\gamma) = 1, q(\beta) = 0\rangle, \text{i.e., the first best.}$$

Note that $P(\gamma) = 1/2 = P(\beta)$ implies the first best, $\langle q(\gamma) = 1, q(\beta) = 0\rangle$ for all cases, so we will not mention it further.

Solution: The PM will use Set 1, i.e., will induce the $\mathcal{II}$ to short sell under the good signal, and buy under the bad signal. She will do this by setting:

$$\left\langle\ q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle \text{ if } P(\gamma) < 1/2 < P(\beta),$$
$$\text{and setting } \left\langle\ q(\gamma) = 1, q(\beta) = \max\left(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\right)\right\rangle \text{ if } P(\gamma) > 1/2 > P(\beta).$$

Remark 3 [Binding constraints and signal probabilities]: Consider why the second constraint (IC_{12}) does not bind where $P(\gamma) > P(\beta)$. Suppose these probabilities, and suppose the policy is at the first-best $\langle q(\gamma) = 1, q(\beta) = 0\rangle$. Here, an $\mathcal{II}(\beta)$ type will want to short sell and deter execution. The UTs believe that the policy is executed "most of the time"; thus deterring execution yields the larger information advantage or "surprise". $\mathcal{II}(\beta)$ can profit from this information advantage in proportion to $\Delta A_1(\gamma)$, as when he short-sells the UTs compensate him for their predicted execution after the *good* signal.

In contrast, if $\mathcal{II}(\beta)$ were to buy and induce execution, this would bring only the smaller $\Delta A_1(\beta)$ in ex-post profit while to buy the asset he would have to compensate the UTs for their expectation of asset gains from execution under the good signal, paying them $P(\gamma)\Delta A_1(\gamma)$.

Now consider why (IC_{12}) *may* bind where $P(\gamma) < P(\beta)$. Here if the policy were first-best the UTs would believe it would be executed less then half of the time; thus *inducing* execution yields the larger surprise, an information advantage of $1 - P(\gamma)$. On the other hand, the (ex-post) gain from inducing execution here is only proportional to $\Delta A_1(\beta)$ but he must pay the UT's for the asset in proportion to $\Delta A_1(\gamma)$; the "asymmetric asset gain" hurts him here. In contrast, by short-selling and deterring execution he is inducing a smaller surprise but will not have to pay for the asymmetric asset gain. These two effects go in the opposite direction, and where the "larger surprise" advantage outweighs the "asymmetric asset gain" cost – i.e., where i.e., where $2P(\gamma) < \frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}$ – then IC_{12} will bind.

1.3.1 Case ii: $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$, we can simplify the two incentive constraints into:

$$q(\beta)\Big(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1 - 2P(\beta)) \le 0, \quad \Big(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1 - 2P(\gamma)) \ge 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$] Then the above is simplified into:

$$q(\beta)\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)} \le 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \le q(\beta)$$

Clearly, the first constraint is not relevant as the PM wants to minimize $q(\beta)$. If $(b) > 0$, then the solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. If $(b) \le 0$, then the second constraint is also irrelevant by the same reason; thus the solution is $< q(\gamma) = 1, q(\beta) = 0 >$. In summary, the solution is:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \left\langle q(\gamma) = 1, q(\beta) = \max\left(0, \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\right)\right\rangle$$

[**Case 2:** $P(\gamma) < \frac{1}{2} < P(\beta)$]. The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad 1 \geq \underbrace{\frac{1 - 2P(\gamma)}{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}}_{(b)} q(\beta)$$

Then the second constraint is irrelevant as PM wants to minimize $q(\beta)$. Note $(a) > 1$, so the solution is:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \left\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle.$$

The contrast between cases (i) and (ii) offers real-world lessons. The choice of which type of behavior to try to induce depends on the relative gains to the asset under the good or bad policy. If the asset does better when the policy is *bad*, policymakers may want to get informed investors to buy only if the policy is bad, promising to execute the policy with some minimum probability.

1.3.2 Case iii: $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$, we can simplify these into:

$$q(\beta)\left(1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\right) - (1 - 2P(\beta)) \geq 0, \quad \left(1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\right) - q(\beta)(1 - 2P(\gamma)) \geq 0$$

[**Case 1:** $P(\gamma) > \frac{1}{2} > P(\beta)$] The above is simplified into:

$$q(\beta)\underbrace{\frac{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1 - 2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}}_{(b)} \leq q(\beta)$$

Note that $(a) > 1$ and $(b) < 0$; thus, the second constraint is irrelevant, and the result follows.

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \left\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle.$$

[**Case 2:** $P(\gamma) < \frac{1}{2} < P(\beta)$] The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)<0} \leq 1, \quad 1 \geq \underbrace{\frac{1-2P(\gamma)}{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}}_{(b)>0} q(\beta)$$

Note that since $(a) < 0$ and $(b) > 0$ hold, both of the constraints are irrelevant, so the solution is:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$

Solution: The PM will use Set 2 and induce the $\mathcal{II}$ to short sell under the good signal, and buy under the bad signal, by setting:

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$
$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \left\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right\rangle.$$

1.3.3 Case iv: $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$

Proposition 8 implies that Set 2 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$, the constraints are simplified into:

$$q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1-2P(\beta)) \geq 0, \quad \Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1-2P(\gamma)) \leq 0$$

[**Case 1:** $P(\gamma) > \frac{1}{2} > P(\beta)$] The above can be simplified into:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)} \geq q(\beta)$$

Note that $(a) > 1$ and $(b) > 1$; thus, the second constraint is irrelevant, and the result follows.

[**Case 2:** $P(\gamma) < \frac{1}{2} < P(\beta)$] then the IC constraints can be written as:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)<1} \leq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)<1} \leq q(\beta)$$

Since $(a) < 1$, the first constraint is irrelevant. If $(b) < 0$, the second constraint is also irrelevant; and the optimal solution is $< q(\gamma) = 1, q(\beta) = 0 >$. If $0 < (b) < 1$, then the optimal solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. So the result follows.

Solution: The PM will use Set 2 and induce the $\mathcal{II}$ to short sell under the good signal and buy under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle,$$

$$\text{and for } P(\gamma) < 1/2 < P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\Big)\Big\rangle$$

1.3.4 Case v: $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$

Proposition 8 implies that Set 1 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6.

Set 1 is:

$$\Delta A_1(\gamma)\Big[q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - q(\gamma)(1-2P(\beta))\Big] \geq 0,$$
$$\Delta A_1(\beta)\Big[q(\gamma)\Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1-2P(\gamma))\Big] \leq 0.$$

Since $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$, we can simplify the above into:

$$q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1-2P(\beta)) \leq 0,\quad \Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1-2P(\gamma)) \geq 0$$

[**Case 1:** $P(\gamma) > \frac{1}{2} > P(\beta)$] Then the above is simplified into:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)} \leq 1,\quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)} \leq q(\beta)$$

Clearly the first constraint is irrelevant as the PM wants to minimize $q(\beta)$.

If $(b) \in (0,1)$, then the optimal solution is $< q(\gamma) = 1, q(\beta) = \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)} >$. If $(b) < 0$, then the second constraint is also irrelevant; thus, the optimal solution is $< q(\gamma) = 1, q(\beta) = 0 >$. In summary, the optimal solution is:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\Big)\Big\rangle.$$

[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$**]** The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)>1} \geq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)>1} \geq q(\beta)$$

Note that $(a) > 1$ and $(b) > 1$. Thus, the second constraint is irrelevant, and the result follows.

Solution: The PM will use Set 1 and induce the $\mathcal{II}$ to buy under the good signal and short sell under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \max\Big(0, \frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}\Big)\Big\rangle, \text{and}$$

$$\text{for } P(\gamma) < 1/2 < P(\beta),\ \Big\langle q(\gamma) = 1, q(\beta) = \frac{1-2P(\beta)}{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Big\rangle.$$

1.3.5 Case vi: $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$

Proposition 8 implies that Set 1 is the relevant one. Also $q(\gamma) = 1$ from Proposition 6. Since $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$, we can simplify these into:

$$q(\beta)\Big(1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}\Big) - (1-2P(\beta)) \geq 0, \quad \Big(1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}\Big) - q(\beta)(1-2P(\gamma)) \geq 0$$

[Case 1: $P(\gamma) > \frac{1}{2} > P(\beta)$**]** Then the above is simplified into:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)} \geq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)} \leq q(\beta)$$

Note that $(a) > 1$ and $(b) < 0$, so that the second constraint is irrelevant, and the result follows from the first constraint.

[Case 2: $P(\gamma) < \frac{1}{2} < P(\beta)$**]** The incentive compatibility constraints are:

$$q(\beta)\underbrace{\frac{1-2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}{1-2P(\beta)}}_{(a)<0} \leq 1, \quad \underbrace{\frac{1-2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1-2P(\gamma)}}_{(b)>1} \geq q(\beta)$$

Note that $(a) < 0$ and $(b) > 1$. Thus, the first constraint is irrelevant, and the result follows.

Solution: The PM will induce the $\mathcal{II}$ to buy under the good signal and short sell under the bad signal, by setting:

$$\text{For } P(\gamma) > 1/2 > P(\beta),\ \left\langle q(\gamma) = 1, q(\beta) = \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\right)\right\rangle.$$

$$\text{For } P(\gamma) < 1/2 < P(\beta),\ \langle q(\gamma) = 1, q(\beta) = 0\rangle.$$

1.4 Short selling or buying is superior to *not buying*

Consider the case of $\Delta A_1(\gamma) > \Delta A_1(\beta)$. $\mathcal{II}(\gamma)$'s payoff when he buys and $\mathcal{II}(\beta)$'s payoff when he short sells are, respectively:

$$q(\gamma)\Delta A_1(\gamma) - (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\beta)[q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)], \text{ and}$$
$$-q(\beta)\Delta A_1(\beta) + (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\gamma)[q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)]$$

As shown in Lemma 5, $q(\gamma) \geq q(\beta)$ must be the case. (Note that the proof for Lemma 5 depends only on the PM's welfare function.)

For case (i) in which $\Delta A_1(\gamma) > \Delta A_1(\beta) > 0$, both of the above payoffs are positive since $q(\gamma) \geq q(\beta)$. The same holds for case (vi) in which $\Delta A_1(\gamma) > 0 > \Delta A_1(\beta)$. For case (v) in which $0 > \Delta A_1(\gamma) > \Delta A_1(\beta)$, suppose $P(\gamma) > P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)$, we derive:

$$\Delta A_1(\gamma) - \frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Delta A_1(\beta) = \frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{1 - 2P(\gamma)} > 0.$$

On the other hand, suppose $P(\gamma) < P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $q(\gamma)\Delta A_1(\gamma) - q(\beta)\Delta A_1(\beta)$, we derive:

$$\Delta A_1(\gamma) - \frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Delta A_1(\beta) = \frac{\Delta A_1(\gamma) - \Delta A_1(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}} > 0.$$

Thus, we have shown that the two payoffs are non-negative when $\Delta A_1(\gamma) > \Delta A_1(\beta)$.

Similarly, consider the case of $\Delta A_1(\gamma) < \Delta A_1(\beta)$. $\mathcal{II}(\gamma)$'s payoff when he short sells and $\mathcal{II}(\beta)$'s payoff when he buys are, respectively:

$$-q(\gamma)\Delta A_1(\gamma) + (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\gamma)[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)], \text{ and}$$
$$q(\beta)\Delta A_1(\beta) - (P(\gamma)q(\gamma)\Delta A(\gamma) + P(\beta)q(\beta)\Delta A(\beta)) = P(\beta)[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$$

For case (ii) in which $\Delta A_1(\beta) > \Delta A_1(\gamma) > 0$, suppose $P(\gamma) > P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$, we derive

$$\frac{1 - 2P(\gamma)\frac{\Delta A_1(\gamma)}{\Delta A_1(\beta)}}{1 - 2P(\gamma)}\Delta A_1(\beta) - \Delta A_1(\gamma) = \frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{1 - 2P(\gamma)} > 0.$$

On the other hand, suppose $P(\gamma) < P(\beta)$. Plugging the optimal $q(\beta)$ and $q(\gamma) = 1$ into $[q(\beta)\Delta A_1(\beta) - q(\gamma)\Delta A_1(\gamma)]$, we derive

$$\frac{1 - 2P(\beta)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}}\Delta A_1(\beta) - \Delta A_1(\gamma) = \frac{\Delta A_1(\beta) - \Delta A_1(\gamma)}{1 - 2P(\beta)\frac{\Delta A_1(\beta)}{\Delta A_1(\gamma)}} > 0.$$

For case (iii) in which $\Delta A_1(\beta) > 0 > \Delta A_1(\gamma)$, the two payoffs are trivially non-negative. The same holds for case (iv) in which $0 > \Delta A_1(\beta) > \Delta A_1(\gamma)$.

Thus we have shown that the payoffs are also non-negative when $\Delta A_1(\gamma) < \Delta A_1(\beta)$.